CONJUGATE ROOT THEOREM

- A polynomial with real coefficients (e.g. $ax^3 + bx^2 + cx + d$, where a, b, c and d are real) that has complex number roots will have them occur in conjugate pairs.
- This is because when you multiply conjugate pairs, the result is a real number.
- Example: $\big(x - (1 + 2i)\big)\big(x - (1 - 2i)\big) = x^2 - 2x + 5$.
- The Conjugate Root Theorem helps us find the roots of cubics which have some complex roots.

CUBIC EQUATIONS WITH TWO COMPLEX ROOTS

Example 1: Find the roots of $x^3 - 12x^2 + 46x - 52$, given that one root is $5 + i$.

- If $5 + i$ is a root, the second root must be $5 - i$ (according to the Conjugate Root Theorem).
- Therefore, two factors of the cubic must be $\big(x - (5 + i)\big)$ and $(x - (5 - i))$.
- The third factor can be written as $(x - c)$.
- $\big(x - (5 + i)\big)\big(x - (5 - i)\big)(x - c) = x^3 - 12x^2 + 46x - 52$
- You can fully expand the LHS and compare to the RHS to solve for c, but we really only need to inspect the constant term:

$$-(5 + i) \times -(5 - i) \times -c = -52$$
$$-(5 + i)(5 - i)c = -52$$
$$-26 \times c = -52$$
$$c = 2$$

- Therefore, the third factor is $(x - 2)$ and so the third root is 2.
- The three roots are $5 + i, 5 - i$, and 2.

Example 2: Use the factor theorem and division to solve the following cubic:

$$x^3 - 2x^2 - 3x + 10 = 0.$$

- Using the factor theorem on factors of -10: 1, 2, 5, 10, -1, -2, -5, 10 we find that f(-2)=0 meaning $(x + 2)$ is a factor.
- Dividing $(x + 2)$ into $x^3 - 2x^2 - 3x + 10$, we get $x^2 + 4x + 5$.

$$\begin{array}{r|l} & x^2 + 4x + 5 \\ \hline x + 2 & x^3 - 2x^2 - 3x + 10 \\ & \underline{x^3 - 2x^2} \\ & \quad 4x^2 - 3x \\ & \quad \underline{4x^2 - 8x} \\ & \qquad 5x + 10 \\ & \qquad \underline{5x + 10} \\ & \qquad\quad 0 \end{array}$$

- Solve $x^2 + 4x + 5 = 0$ using the quadratic formula:

$$x^2 + 4x + 5 = 0$$
$$x = \frac{-4 \pm \sqrt{16 - 20}}{2}$$
$$x = \frac{-4 \pm \sqrt{-4}}{2}$$
$$x = \frac{-4 \pm \sqrt{4i^2}}{2}$$
$$x = \frac{-4 \pm \sqrt{4}i}{2}$$
$$x = -2 \pm i$$

- Therefore the roots for the cubic $x^3 - 2x^2 - 3x - 10 = 0$ are -2, $-2 + i$, and $-2 - i$.

Delta Ex 31.6 pg 290 Q3 – 7

Extension Ex 31.5 pg 288